Practice 13

Topic: ***Research on stability by the second (direct) method of Lyapunov, using Schultz-Gibson's method***

Example Let system dynamic equations in the state-space with are set the following look:

$$\begin{cases} \dot{x}_1 = -x_1 x_2{}^2 + x_2 \\ \dot{x}_2 = -x_1{}^3 \end{cases} ; \quad x \in R^2.$$

Investigate on stability by the second (direct) method of Lyapunov, using Schultz-Gibson's method.

Algorithm and solution

1. The gradient of function of Lyapunov in the form of Schultz – Gibson as follows registers:

$$\nabla V = \begin{vmatrix} \alpha_{11}x_1 + \alpha_{12}x_2 \\ \alpha_{21}x_1 + \alpha_{22}x_2 \end{vmatrix} = \begin{vmatrix} \nabla V_1 \\ \nabla V_2 \end{vmatrix}. \qquad (**)$$

2. The full derivative of function of Lyapunov is defined:

$$\frac{dV}{dt} = \nabla V^T \dot{x} = \nabla V_1 \dot{x}_1 + \nabla V_2 \dot{x}_2 = (\alpha_{11}x_1 + \alpha_{12}x_2)(-x_1x_2{}^2 + x_2) - (\alpha_{21}x_1 + \alpha_{22}x_2)x_1{}^3 =$$

$$= -\alpha_{11}x_1{}^2x_2{}^2 + \alpha_{11}x_1x_2 - \alpha_{12}x_1x_2{}^3 + \alpha_{12}x_2{}^2 - \alpha_{21}x_1{}^4 - \alpha_{22}x_1{}^3x_2 < 0$$

3. It is necessary to define such α_{ij} values that the full derivative of function of Lyapunov was strictly negative, i.e. $\dot{V}(x) < 0$.

Let $\alpha_{11}=1$; $\alpha_{12}=-\alpha_{21}$; $\alpha_{12}=-1$; $\alpha_{21}=1$.

Then

$$\frac{dV}{dt} = -x_1{}^2x_2{}^2 + x_1x_2(1 + x_2{}^2) - x_2{}^2 - x_1{}^4 - \alpha_{22}x_1{}^3x_2 < 0.$$

From here

$$\alpha_{22} = \frac{1 + x_2{}^2}{x_1{}^2} = \frac{1}{x_1{}^2} + \frac{x_2{}^2}{x_1{}^2}.$$

At such choice of coefficients α_{ij} we will receive the following:

$$\frac{dV}{dt} = -x_1^2x_2^2 - x_1^2 - x_2^4 < 0.$$

4. It is necessary to define Lyapunov's function and to be convinced that *V(x)>0.*

Define Lyapunov's function by *V(x)*, we will substitute the found coefficients in a gradient (**):

$$\nabla V = \begin{vmatrix} x_1 - x_2 \\ x_1 + \dfrac{x_2}{x_1^2} + \dfrac{x_2^3}{x_1^2} \end{vmatrix} = \begin{vmatrix} \nabla V_1 \\ \nabla V_2 \end{vmatrix}.$$

Having used expression (*), we will obtain Lyapunov's function of the following look:

$$V(x) = \int_0^x \nabla V^T dx = \int_0^{x_1} \nabla V_1(\xi_1, 0)d\xi_1 + \int_0^{x_2} \nabla V_2(x_1, \xi_2)d\xi_2 =$$

$$= \int_0^{x_1} \xi_1 d\xi_1 + \int_0^{x_2} (x_1 + \frac{\xi_2}{x_1^2} + \frac{\xi_2^3}{x_1^2})d\xi_2 = \frac{1}{2}x_1^2 + x_1x_2 + \frac{x_2^2}{2x_1^2} + \frac{x_2^4}{4x_1^2} > 0.$$

Hence, the initial system is *asymptotically stable according to Lyapunov.*

It is reasonable to emphasize the basic conclusions once more:

a) $\begin{cases} V > 0 \\ \dot{V} < 0 \end{cases}$ – function must be one of fixed positive-sign; – differential must be one of strictly negative-sign, which provides *asymptomatically stability* to the system according to Lyapunov.

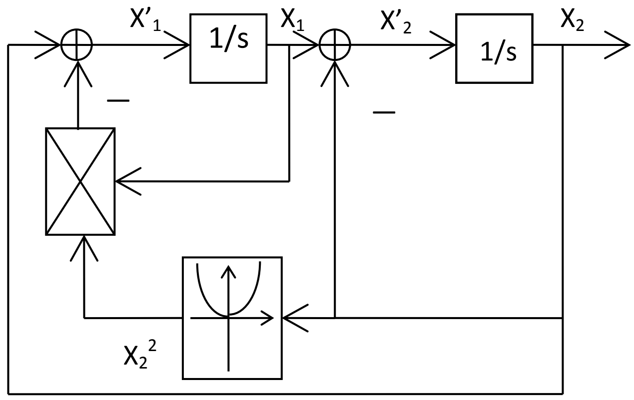
b) $\begin{cases} V > 0 \\ \dot{V} \le 0 \end{cases}$ – function should be one of fixed positive-sign; – its full differential may be one of fixed negative-sign, which provides *stability* to the system according to Lyapunov.

c) $\begin{cases} V > 0 \\ \dot{V} > 0 \end{cases}$ – the both functions are of fixed positive-sign, which provides *instability* to the system according to Lyapunov.

Task: Obtain dynamic equations of nonlinear ACS in the state-space according to the set scheme (by variants). Investigate on stability by the second (direct) method of Lyapunov, using Schultz-Gibson's method.

Variants:

1)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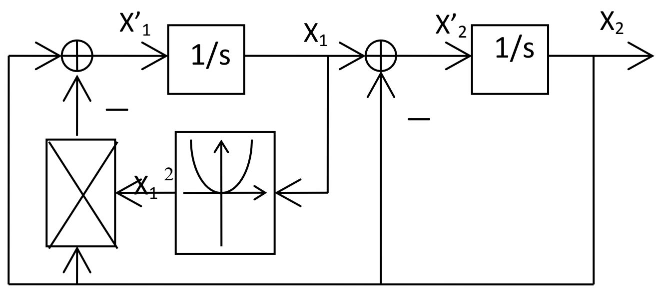
X1
X’2
1/s
X2
—
—
X2 2

2)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X2
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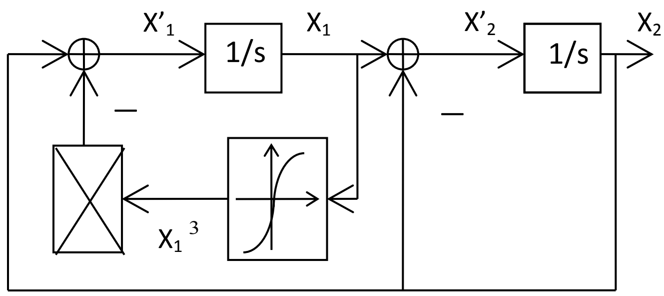
—
X1 2

3)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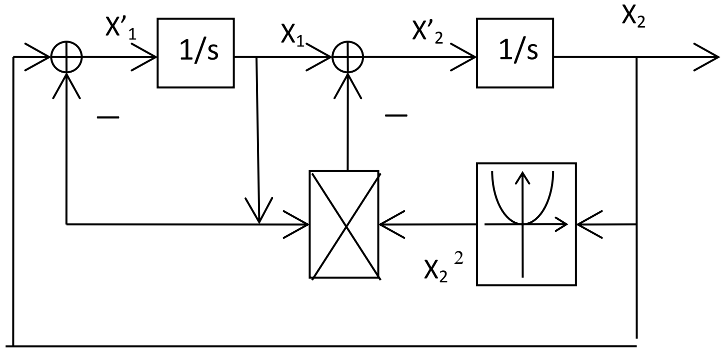
—
X1 3

4)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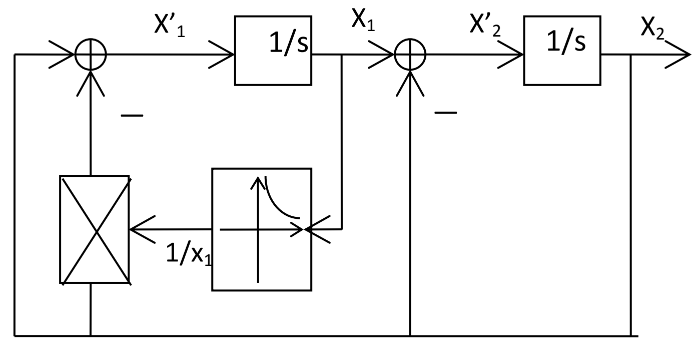
X1
X’2
1/s
X2
—
—
X2 2

5)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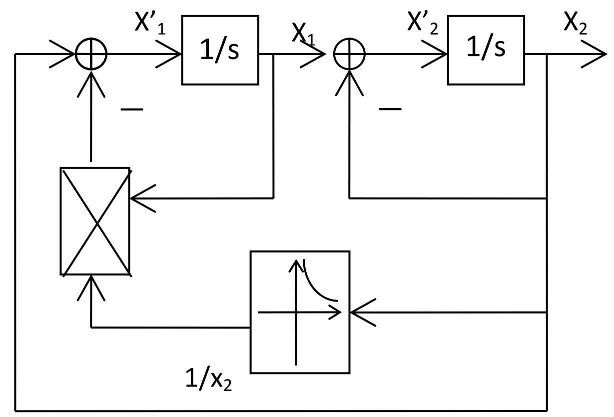
X1
X'2
1/s
X2
1/x1

6)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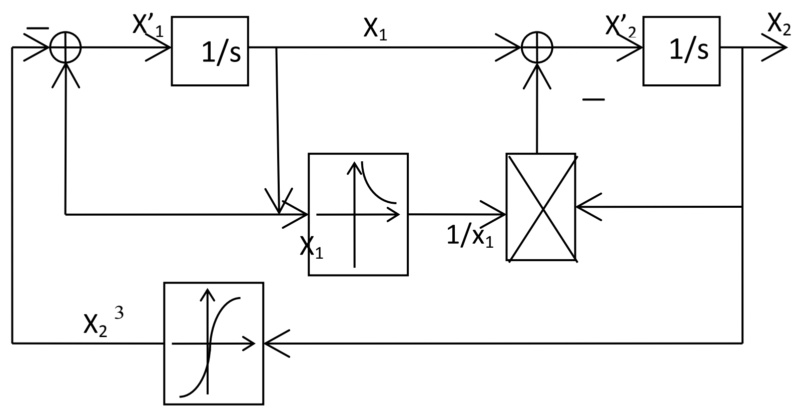
X1
X'2
1/s
X2
1/x2

7)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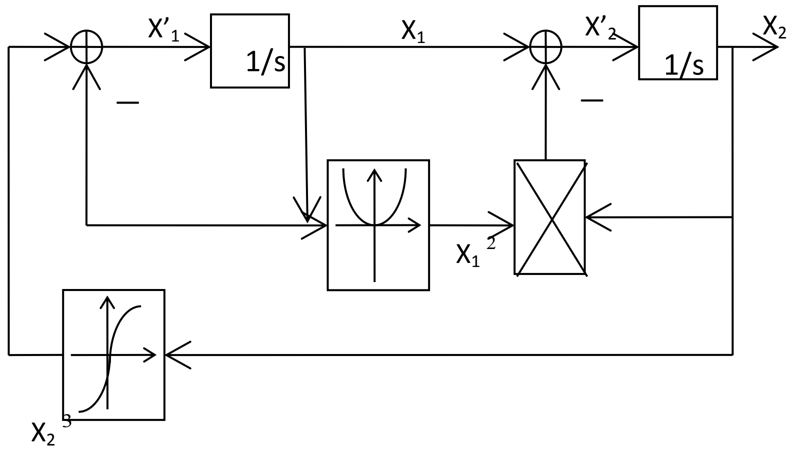
X1
X'2
1/s
X2
X1
1/x1
X2 3

8)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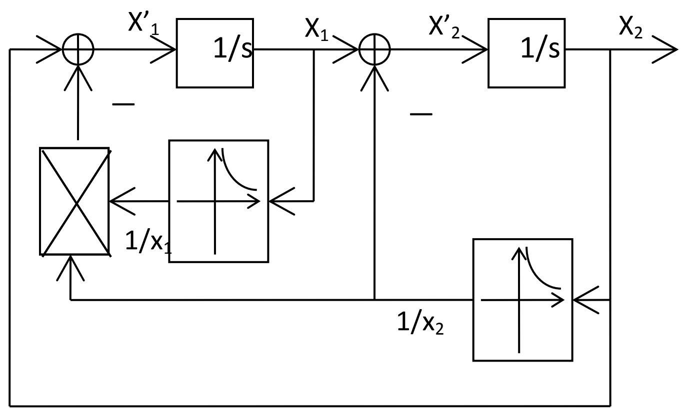
X1
X’2
1/s
X2
X1 2
X2 3

9)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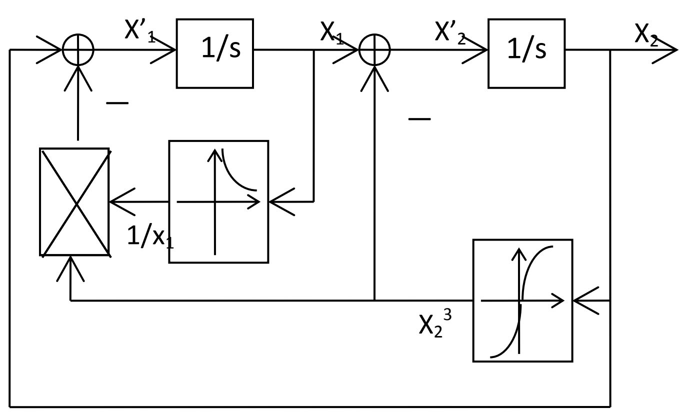
X1
X’2
1/s
X2
1/x1
1/x2

10)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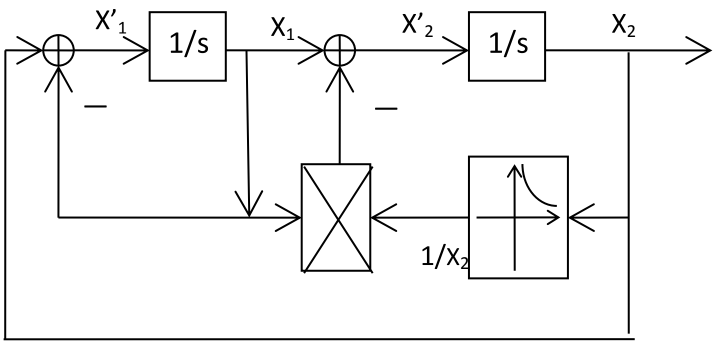
X1
X’2
1/s
X2
1/x1
X2 3

11)



12)